International Journal of Foreign Trade and International Business

E-ISSN: 2663-3159
P-ISSN: 2663-3140
Impact Factor: RJIF 5.22
www.foreigntradejournal.com
IJFTIB 2023; 5(1): 21-23
Received: 16-10-2022
Accepted: 28-11-2022

EL Pankratov
Nizhny Novgorod State University, 23 Gagarin Avenue, Nizhny Novgorod, 603950, Russia

Corresponding Author:
EL Pankratov
Nizhny Novgorod State University, 23 Gagarin Avenue, Nizhny Novgorod, 603950, Russia

A model for description of random economic processes

EL Pankratov

DOI: https://doi.org/10.33545/26633140.2023.v5.i1a.83

Abstract
In this paper we introduce a model for prognosis of random economical processes. The approach gives a possibility to make prognosis of the considered processes during changing of economical situation. Also we introduce an analytical approach for analysis of the introduced model.

Keywords: Random economical processes; approach for prognosis

Introduction
Economic systems are subject to the influence of a large number of uncontrollable external factors (weather conditions, foreign policy, social factors, deliberate distortion and concealment of information for the purpose of economic sabotage, etc.) [1-5]. In this situation, the system parameters can take random values. This paper considers a model for prognosis of processes in economic systems with random values of parameters. We also introduce an analytical approach for analysis of the model.

Method of solution
Let us consider a random process $\eta(t)$. Let us fix moments of time t_1, t_2, ..., t_n. Probability of obtaining of value $\eta(t_n)$ in the following interval $\eta(t_n)\in[x_n,x_n+d\,x_n]$ if moment t_1 random process assumes value x_1, if moment t_2 random process assumes value x_2, ..., if moment t_{n-1} random process assumes value x_{n-1} is equal to the product of the conditional probability density and the length of the interval: $P=W(x_1,t_1;\, x_2,t_2;\, ...;\, x_{n-1},t_{n-1}|\, x_n,t_n)\, d\, x_n$. In the common case the considered probability depends on all values x_n,t_n. We consider a model with higher exactness. At the same time analysis of the model is harder. In this situation it is attracted an interests model with compromise: the model gives a possibility to simplify prognosis of processes with maximal saving of adequateness of description. As one of most simple models it could be consider Markov process: we consider only one previous value x_{n-1} during analysis of the considered process. Probability density of the Markov process describes by the Focker -Planck equation [6].

$$\frac{\partial\, W(x,t)}{\partial t}=\frac{\partial^2\left[D(x,t)\,W(x,t)\right]}{\partial\, x^2}-\frac{\partial\left[K(x,t)\,W(x,t)\right]}{\partial\, x}, \tag{1}$$

Where $D(x,t)$ is the diffusion coefficient, $K(x,t)$ is the drift coefficient. To illustrate the introduced method of solution of the equation (1) we consider the following boundary and initial conditions

$$W(0,t)=0;\; W(L,t)=0;\; W(x,0)=f(x). \tag{2}$$

At variable values of coefficients of diffusion and drift one can obtain exact solution of the equation (1) only for few dependences of the considered coefficients on variables x and t. In this situation we consider an approach to obtain approximate solution of the considered equations. To describe the approach we transform coefficients $D(x,t)$ and $K(x,t)$ in the following forms

$$D(x,t)=D_0[1+\varepsilon\cdot g(x,t)],\; K(x,t)=K_0[1+\xi\cdot h(x,t)], \tag{3}$$

Where $|g(x,t)|\le 1$, $|h(x,t)|\le 1$, $0\le \varepsilon <1$, $0\le \xi <1$, D_0 and K_0 are the average values of the considered coefficients. Next we will determine solution of the equation (1) as the following series

$$W(x,t)=\sum_{k=0}^{\infty}\varepsilon^k\sum_{l=0}^{\infty}\xi^l W_{kl}(x,t). \tag{4}$$

Substitution of the above series and relations (3) into the equation (1) leads to the following result

$$\sum_{k=0}^{\infty}\varepsilon^k\sum_{l=0}^{\infty}\xi^l\frac{\partial W_{kl}(x,t)}{\partial t}=D_0\sum_{k=0}^{\infty}\varepsilon^k\sum_{l=0}^{\infty}\xi^l\frac{\partial^2\{[1+\varepsilon\cdot g(x,t)]W_{kl}(x,t)\}}{\partial x^2}-$$

$$-K_0\sum_{k=0}^{\infty}\varepsilon^k\sum_{l=0}^{\infty}\xi^l\frac{\partial\{[1+\xi\cdot h(x,t)]W_{kl}(x,t)\}}{\partial x}. \tag{5}$$

Grouping the coefficients for the same powers of the parameters ε and ξ makes it possible to obtain equations for the functions $W_{kl}(x,t)$

$$\frac{\partial W_{kl}(x,t)}{\partial t}=D_0\frac{\partial^2 W_{kl}(x,t)}{\partial x^2}+D_0\frac{\partial^2[g(x,t)\cdot W_{k-1l}(x,t)]}{\partial x^2}-$$

$$-K_0\frac{\partial W_{kl}(x,t)}{\partial x}-K_0\frac{\partial[h(x,t)\cdot W_{kl-1}(x,t)]}{\partial x},\ k\ge 0,\ l\ge 0. \tag{6}$$

Boundary and initial conditions for the functions $W_{kl}(x,t)$ could be written in the form

$W_{kl}(0,t)=0$, $W_{kl}(L,t)=0$, $k\ge 0$, $l\ge 0$; $W_{00}(x,0)=f(x)$, $W_{kl}(x,t)=0$, $k\ge 1$, $l\ge 1$. (7)

Equations (6) with conditions (7) could be solved, for example, by the Fourier approach. The obtained solutions could be written as

$$W_{00}(x,t)=\sum_{n=0}^{\infty}e^{-t\frac{\pi^2n^2D_0^2+K_0^2L^2}{4D_0L^2}}\sin\left(\frac{\pi n x}{L}\right)\int_0^L f(x)\sin\left(\frac{\pi n x}{L}\right)dx$$

$$W_{10}(x,t)=\frac{1}{2}\sum_{m=0}^{\infty}e^{-t\frac{\pi^2m^2D_0^2+K_0^2L^2}{4D_0L^2}}\sin\left(\frac{\pi m x}{L}\right)\sum_{n=0}^{\infty}e^{-t\frac{\pi^2n^2D_0^2+K_0^2L^2}{4D_0L^2}}\int_0^L f(x)\sin\left(\frac{\pi n x}{L}\right)dx\times$$

$$\times\int_0^L g(x,t)\left\{\cos\left[\frac{\pi(m-n)x}{L}\right]-\cos\left[\frac{\pi(m+n)x}{L}\right]\right\}dx$$

$$W_{01}(x,t)=-\frac{1}{2}\sum_{m=0}^{\infty}e^{-t\frac{\pi^2m^2D_0^2+K_0^2L^2}{4D_0L^2}}\sin\left(\frac{\pi m x}{L}\right)\sum_{n=0}^{\infty}e^{-t\frac{\pi^2n^2D_0^2+K_0^2L^2}{4D_0L^2}}\int_0^L f(x)\sin\left(\frac{\pi n x}{L}\right)dx\times$$

$$\times\int_0^L h(x,t)\left\{\cos\left[\frac{\pi(m-n)x}{L}\right]-\cos\left[\frac{\pi(m+n)x}{L}\right]\right\}dx$$

$$W_{11}(x,t)=\frac{1}{2}\sum_{m=0}^{\infty}e^{-t\frac{\pi^2m^2D_0^2+K_0^2L^2}{4D_0L^2}}\sin\left(\frac{\pi m x}{L}\right)\sum_{n=0}^{\infty}e^{-t\frac{\pi^2n^2D_0^2+K_0^2L^2}{4D_0L^2}}\int_0^L g(x,t)\left\{\cos\left[\frac{\pi(m-n)x}{L}\right]-\cos\left[\frac{\pi(m+n)x}{L}\right]\right\}dx\times$$

$$\times\int_0^L f(x)\sin\left(\frac{\pi n x}{L}\right)dx-\frac{1}{2}\sum_{k=0}^{\infty}e^{-t\frac{\pi^2k^2D_0^2+K_0^2L^2}{4D_0L^2}}\sum_{l=0}^{\infty}e^{-t\frac{\pi^2l^2D_0^2+K_0^2L^2}{4D_0L^2}}\int_0^L h(x,t)\left\{\cos\left[\frac{\pi(k-l)x}{L}\right]-\cos\left[\frac{\pi(k+l)x}{L}\right]\right\}dx\times$$

$$\times\sin\left(\frac{\pi k x}{L}\right)\int_0^L f(x)\sin\left(\frac{\pi l x}{L}\right)dx$$

The features of this method are

1. The ability to solve equation (1) for any values of the parameters ε and ξ due to the impossibility of taking negative values by the coefficients $D(x,t)$ and $K(x,t)$: series (4) is convergent. At the same time, this series has an infinite number of terms. In this situation, series (4) cannot be used in full, which leads to the need to approximate the solution of equation (1) by the finite sum of series (4);
2. Using of the considered method of solution with discontinuous coefficients $D(x,t)$ and $K(x,t)$ makes it possible to obtain a solution to equation (1) without using the matching of solutions at discontinuity points [8, 9]. The feature gives a possibility to reduce the volume of transformations [8, 9].

Conclusion

We introduce model for prognosis of random economical processes. Based on the approach we have a possibility to make prognosis of the above processes in the framework of changing of economical situation. Also we introduce an analytical approach for analysis of the introduced model.

References

1. Nathaphan S, Chunhachinda P. Economics research international; c2010. Article ID 340181 (2010).
2. Prat J. Mathematics and financial economics. 2015;9(3):169-193.
3. Ch. W, Wu W, Zh Hung. Economics research international; c2010. Article ID 678927 (2010).
4. St. Villeneuve, Warin X. Mathematics and financial economics. 2014;8(2):193-227.
5. Ch. Mertzanis. Studies in economics and finance. 2017;34(1):82-104.
6. Risken H. The Fokker-Planck equation: methods of solution and applications (Berlin: Springer-Verlag); c1984.
7. Tikhonov AN, Samarsky AA. The equations of mathematical physics (Moscow: Science); c1972.
8. Pankratov EL. Physical Review B. 2005;72(7):075201-075208.
9. EL Pankratov, Bulaeva EA. Reviews in theoretical science. 2013;1(1):58-82.