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## On prognosis of activity of industrial enterprise

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### Abstract

In this paper we present a model for forecasting the activity of industrial enterprises based on solving a differential equation. This model gives a possibility to do prognosis of activity of industrial enterprises taking into account changing of the volume of produced products, as well as taking into account changes in its shipment to the domestic market and export deliveries. An analytical approach to solve the differential equation has been introduced.

**Keywords:** Industrial enterprises; prognosis of activity; analytical approach for prognosis

### Introduction

One of the most important factors, which determines increasing of competitiveness and the successful operation of industrial enterprises, is the development of all spheres of its activity [1, 2]. In this situation enterprises must develop effective methods and adhere to the strategic concept of innovation development [3-5].

In this paper we introduce a model for prognosis of the production activity of enterprises with account the changing of volume of products produced in time. We also taking into account the change of its shipment to the domestic market and export deliveries. Based on the model we analyzed changing of activity of enterprises. This analysis based on the solution of an ordinary differential equation. In this paper we also introduce an analytical approach to analyze the considered model.

### Method of solution

We will analyze the industrial activity of enterprises taking into account the shipment of the products produced by analyzing the following initial problem

$$\frac{dY(t)}{dt} = a_0 \cdot Y(t) + b_0 \cdot Y(t) + a_1 \cdot Y(t - \tau_1) + b_1 \cdot Y(t - \tau_1) + a_2 \cdot Y(t - \tau_2) + b_2 \cdot Y(t - \tau_2) + d(t) \quad (1)$$

with initial condition

$$Y(0) = 0 \quad (2)$$

Here  $Y(t)$  is the volume of shipped products;  $\tau_i$  is the delays (i.e.  $Y(t - \tau_i)$  is the volume of products shipped with delay  $\tau_i$ );  $a_i$  and  $b_i$  are the model coefficients, determined by empirical data ( $a_i$  - model coefficients corresponding to the shipment of products to the domestic market;  $b_i$  - coefficients of the model corresponding to the shipment of products for export);  $d(t)$  is the increasing of volume of product due to its production at one unit of time. The considered delays could be obtained by limited production speed, limited volume of transport units, etc. Framework of this paper, we consider the simplest case of one delay. Within the considered approach, more delays can be taken into account. Then we transform equation (1) to the integral form (1a)

$$Y(t) = (a_0 + b_0) \cdot \int_0^t Y(\theta) d\theta + (a_1 + b_1) \cdot \int_0^t Y(\theta - \tau_1) d\theta +$$

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$$+ (a_2 + b_2) \cdot \int_0^t Y(\theta - \tau_2) d\theta + \int_0^t d(\theta) d\theta \tag{1a}$$

We solve Eq. (1) by using the Bubnov-Galerkin approach [6]. Framework the approach, we will determine the solution of the equation in the form of a sum of exponential functions

$$Y(t) = f_0 e^{-(a_0+b_0)t} + f_1 e^{-(a_1+b_1)t} + f_2 e^{-(a_2+b_2)t} \tag{3}$$

These functions are the most natural solutions of equation (1) framework the classical theory of differential equations [7]. Substitution of the function (3) to the Eq. (1) leads to the following result

$$\begin{aligned} f_0 e^{-(a_0+b_0)t} + f_1 e^{-(a_1+b_1)t} + f_2 e^{-(a_2+b_2)t} = & -(a_0 + b_0) \cdot \left[ \frac{f_0}{a_0 + b_0} e^{-(a_0+b_0)t} + \frac{f_1}{a_1 + b_1} e^{-(a_1+b_1)t} + \right. \\ & \left. + \frac{f_2}{a_2 + b_2} e^{-(a_2+b_2)t} \right] - (a_1 + b_1) e^{(a_1+b_1)\tau_1} \cdot \left[ \frac{f_0}{a_0 + b_0} e^{-(a_0+b_0)t} + \frac{f_1}{a_1 + b_1} e^{-(a_1+b_1)t} + \frac{f_2}{a_2 + b_2} e^{-(a_2+b_2)t} \right] - \\ & - (a_2 + b_2) e^{(a_2+b_2)\tau_2} \cdot \left[ \frac{f_0}{a_0 + b_0} e^{-(a_0+b_0)t} + \frac{f_1}{a_1 + b_1} e^{-(a_1+b_1)t} + \frac{f_2}{a_2 + b_2} e^{-(a_2+b_2)t} \right] \end{aligned} \tag{4}$$

Framework the Bubnov-Galerkin approach we multiply left and right sides of the relation (4a) alternately on  $e^{-(a_0+b_0)t}$ ,  $e^{-(a_1+b_1)t}$  or  $e^{-(a_2+b_2)t}$  and integrating from 0 to  $\infty$ . The procedure gives a possibility to obtain equations for calculation of coefficients  $f_0, f_1$  and  $f_2$

$$\begin{aligned} \frac{f_0}{2(a_0 + b_0)} + \frac{f_1}{a_0 + b_0 + a_1 + b_1} + \frac{f_2}{a_0 + b_0 + a_2 + b_2} = & -(a_0 + b_0) \cdot \left[ \frac{f_0}{2(a_0 + b_0)^2} + \right. \\ & \left. + \frac{f_1}{(a_1 + b_1)(a_0 + b_0 + a_1 + b_1)} + \frac{f_2}{(a_2 + b_2)(a_0 + b_0 + a_2 + b_2)} \right] - (a_1 + b_1) e^{(a_1+b_1)\tau_1} \cdot \\ & \cdot \left[ \frac{f_0}{2(a_0 + b_0)^2} + \frac{f_1}{(a_1 + b_1)(a_0 + b_0 + a_1 + b_1)} + \frac{f_2}{(a_2 + b_2)(a_0 + b_0 + a_2 + b_2)} \right] + \\ & - (a_2 + b_2) \cdot \left[ \frac{f_0}{2(a_0 + b_0)^2} + \frac{f_1}{(a_1 + b_1)(a_0 + b_0 + a_1 + b_1)} + \frac{f_2}{(a_2 + b_2)(a_0 + b_0 + a_2 + b_2)} \right] \cdot \\ & \cdot e^{(a_2+b_2)\tau_2} - \int_0^\infty e^{-(a_0+b_0)t} d(t) dt \\ \frac{f_0}{a_0 + b_0 + a_1 + b_1} + \frac{f_1}{2(a_1 + b_1)} + \frac{f_2}{a_1 + b_1 + a_2 + b_2} = & -(a_0 + b_0) \cdot \left[ \frac{f_0}{(a_0 + b_0)(a_0 + b_0 + a_1 + b_1)} + \right. \\ & \left. + \frac{f_1}{2(a_1 + b_1)^2} + \frac{f_2}{(a_2 + b_2)(a_1 + b_1 + a_2 + b_2)} \right] - (a_1 + b_1) e^{(a_1+b_1)\tau_1} \cdot \left[ \frac{f_0}{(a_0 + b_0)(a_0 + b_0 + a_1 + b_1)} + \right. \\ & \left. + \frac{f_1}{2(a_1 + b_1)^2} + \frac{f_2}{(a_2 + b_2)(a_1 + b_1 + a_2 + b_2)} \right] - (a_2 + b_2) e^{(a_2+b_2)\tau_2} \cdot \left[ \frac{f_0}{(a_0 + b_0)(a_0 + b_0 + a_1 + b_1)} + \right. \\ & \left. + \frac{f_1}{2(a_1 + b_1)^2} + \frac{f_2}{(a_2 + b_2)(a_1 + b_1 + a_2 + b_2)} \right] - \int_0^\infty e^{-(a_1+b_1)t} d(t) dt \\ \frac{f_0}{a_0 + b_0 + a_2 + b_2} + \frac{f_1}{a_1 + b_1 + a_2 + b_2} + \frac{f_2}{2(a_2 + b_2)} = & -(a_0 + b_0) \cdot \left[ \frac{f_0}{(a_0 + b_0)(a_0 + b_0 + a_2 + b_2)} + \right. \\ & \left. + \frac{f_1}{(a_1 + b_1)(a_1 + b_1 + a_2 + b_2)} + \frac{f_2}{2(a_2 + b_2)^2} \right] - (a_1 + b_1) e^{(a_1+b_1)\tau_1} \cdot \left[ \frac{f_0}{(a_0 + b_0)(a_0 + b_0 + a_2 + b_2)} + \right. \\ & \left. + \frac{f_1}{(a_1 + b_1)(a_1 + b_1 + a_2 + b_2)} + \frac{f_2}{2(a_2 + b_2)^2} \right] - (a_2 + b_2) e^{(a_2+b_2)\tau_2} \cdot \left[ \frac{f_0}{(a_0 + b_0)(a_0 + b_0 + a_1 + b_1)} + \right. \\ & \left. + \frac{f_1}{(a_1 + b_1)(a_1 + b_1 + a_2 + b_2)} + \frac{f_2}{2(a_2 + b_2)^2} \right] \end{aligned}$$

$$+ \frac{f_1}{(a_1 + b_1)(a_1 + b_1 + a_2 + b_2)} + \frac{f_2}{2(a_2 + b_2)^2} \left] - \int_0^\infty e^{-(a_2 + b_2)t} d(t) dt$$

Solution of the system of equations (see, for example, [7]) leads to the following result

$$\begin{aligned}
 f_0 &= \left\{ (\beta_3 \gamma_2 - \beta_2 \gamma_3) \int_0^\infty e^{-(a_0 + b_0)t} d(t) dt - \beta_1 \left[ \gamma_2 \int_0^\infty e^{-(a_2 + b_2)t} d(t) dt - \gamma_3 \int_0^\infty e^{-(a_1 + b_1)t} d(t) dt \right] + \right. \\
 &+ \gamma_1 \left[ \beta_2 \int_0^\infty e^{-(a_2 + b_2)t} d(t) dt - \beta_3 \int_0^\infty e^{-(a_1 + b_1)t} d(t) dt \right] \left. \right\} / \left[ \alpha_1 (\beta_2 \gamma_3 - \beta_3 \gamma_2) - \beta_1 (\alpha_2 \gamma_3 - \alpha_3 \gamma_2) + \right. \\
 &+ \gamma_1 (\alpha_2 \beta_3 - \alpha_3 \beta_2) \left. \right], \\
 f_1 &= \left\{ (\alpha_2 \gamma_3 - \alpha_3 \gamma_2) \int_0^\infty e^{-(a_0 + b_0)t} d(t) dt - \alpha_1 \left[ \gamma_3 \int_0^\infty e^{-(a_1 + b_1)t} d(t) dt - \gamma_2 \int_0^\infty e^{-(a_2 + b_2)t} d(t) dt \right] - \right. \\
 &- \gamma_1 \left[ \alpha_2 \int_0^\infty e^{-(a_2 + b_2)t} d(t) dt - \alpha_3 \int_0^\infty e^{-(a_1 + b_1)t} d(t) dt \right] \left. \right\} / \left[ \alpha_1 (\beta_2 \gamma_3 - \beta_3 \gamma_2) - \beta_1 (\alpha_2 \gamma_3 - \alpha_3 \gamma_2) + \right. \\
 &+ \gamma_1 (\alpha_2 \beta_3 - \alpha_3 \beta_2) \left. \right], \\
 f_2 &= \left\{ \beta_1 \left[ \alpha_2 \int_0^\infty e^{-(a_2 + b_2)t} d(t) dt - \alpha_3 \int_0^\infty e^{-(a_1 + b_1)t} d(t) dt \right] - \alpha_1 \left[ \beta_2 \int_0^\infty e^{-(a_2 + b_2)t} d(t) dt - \right. \right. \\
 &- \left. \left. \beta_3 \int_0^\infty e^{-(a_1 + b_1)t} d(t) dt \right] - (\alpha_2 \beta_3 - \alpha_3 \beta_2) \int_0^\infty e^{-(a_0 + b_0)t} d(t) dt \right\} / \left[ \alpha_1 (\beta_2 \gamma_3 - \beta_3 \gamma_2) - \right. \\
 &- \left. \beta_1 (\alpha_2 \gamma_3 - \alpha_3 \gamma_2) + \gamma_1 (\alpha_2 \beta_3 - \alpha_3 \beta_2) \right],
 \end{aligned} \tag{5}$$

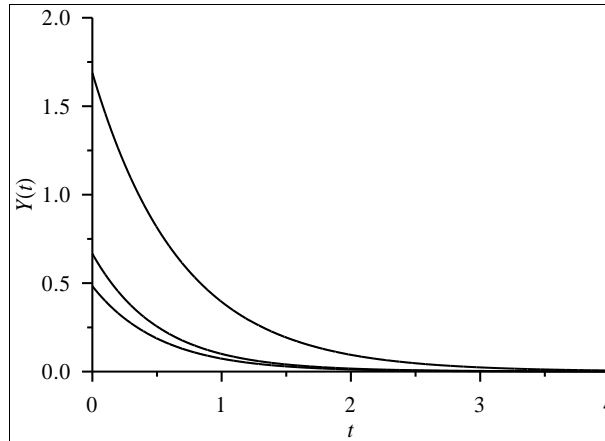
Where

$$\begin{aligned}
 \alpha_1 &= \frac{1}{2(a_0 + b_0)} \left[ 2 + e^{(a_1 + b_1) \tau_1} \frac{a_1 + b_1}{a_0 + b_0} + e^{(a_2 + b_2) \tau_2} \frac{a_2 + b_2}{a_0 + b_0} \right], \\
 \beta_1 &= \frac{1}{a_0 + b_0 + a_1 + b_1} \times \\
 &\times \left\{ 1 + \frac{a_0 + b_0}{a_1 + b_1} + \frac{a_0 + b_0}{a_2 + b_2} + \exp[(a_1 + b_1) \tau_1] + \exp[(a_2 + b_2) \tau_2] \frac{a_2 + b_2}{a_1 + b_1} \right\}, \\
 \gamma_1 &= \frac{1}{a_0 + b_0 + a_2 + b_2} \left\{ 1 + \exp[(a_1 + b_1) \tau_1] \frac{a_1 + b_1}{a_2 + b_2} + \exp[(a_2 + b_2) \tau_2] \right\}, \\
 \alpha_2 &= \frac{1}{a_0 + b_0 + a_1 + b_1} \left[ 2 + \frac{a_1 + b_1}{a_0 + b_0} e^{(a_1 + b_1) \tau_1} + \frac{a_2 + b_2}{a_0 + b_0} e^{(a_2 + b_2) \tau_2} \right], \quad \beta_2 = \frac{1}{2(a_1 + b_1)} \times \\
 &\times \left[ 1 + \frac{a_0 + b_0}{a_1 + b_1} + e^{(a_1 + b_1) \tau_1} + \frac{a_2 + b_2}{a_1 + b_1} e^{(a_2 + b_2) \tau_2} \right], \quad \gamma_2 = \frac{1}{a_1 + b_1 + a_2 + b_2} \left[ 1 + \frac{a_1 + b_1}{a_2 + b_2} e^{(a_1 + b_1) \tau_1} + \right. \\
 &+ \left. \frac{a_0 + b_0}{a_2 + b_2} + e^{(a_2 + b_2) \tau_2} \right], \quad \alpha_3 = \frac{1}{a_0 + b_0 + a_2 + b_2} \left[ 2 + \frac{a_1 + b_1}{a_0 + b_0} e^{(a_1 + b_1) \tau_1} + \frac{a_2 + b_2}{a_0 + b_0} e^{(a_2 + b_2) \tau_2} \right], \\
 \beta_3 &= \frac{1}{a_1 + b_1 + a_2 + b_2} \left[ 1 + e^{(a_1 + b_1) \tau_1} + \frac{a_0 + b_0}{a_1 + b_1} + \frac{a_2 + b_2}{a_1 + b_1} e^{(a_2 + b_2) \tau_2} \right], \quad \gamma_3 = \frac{1}{2(a_2 + b_2)} \times
 \end{aligned}$$

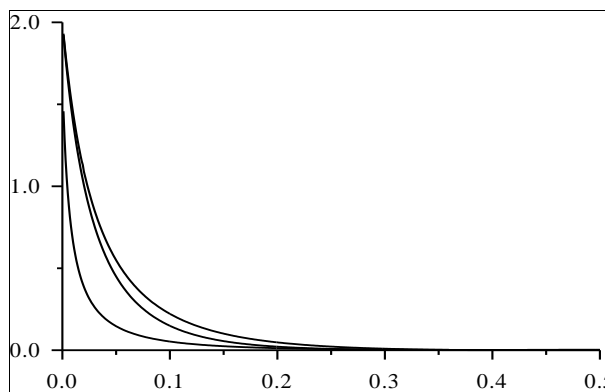
$$\times \left[ 1 + \frac{a_0 + b_0}{a_2 + b_2} + \frac{a_1 + b_1}{a_2 + b_2} e^{(a_1 + b_1)\tau_1} + e^{(a_2 + b_2)\tau_2} \right]$$

**Discussion**

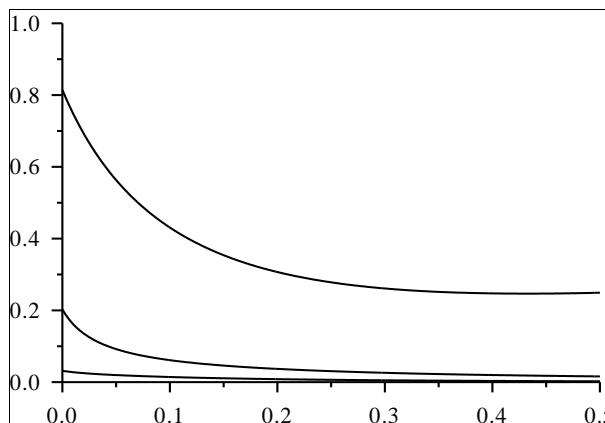
We will use relations obtained in the previous section to analyze of the activity of enterprises, taking into account changes in the volume of manufactured products, as well as taking into account changes in its shipment to the domestic market and export deliveries. In Fig. 1 shows the typical dependences of the volume of products shipped from time for different values of the parameters  $a_i$  and  $b_i$  with a constant increasing in the volume of production  $d_0$ . In Fig. 2 shows the typical dependences of the volume of the shipped product on the value of the parameter  $a$  for various values of the parameters  $b_i$  at different times  $t$  and with a constant increasing of the volume of production  $d_0$ . Analogous are the dependences of the volume of the product shipped on the parameters  $b_i$  (see Figure 3). Dependences of the volume of products shipped from delays  $\tau_i$  are also decreasing functions (see Figure 4). Thus, with the acceleration of export of shipped products, its quantity decreases and it is necessary to increase its production. With decreasing the parameters  $a_i$  and  $b_i$ , as well as delays,  $\tau_i$  the volume of the shipped products increases, which corresponds to the accumulation of output. In this situation, the curves in Figures 1-4 will be increasing.



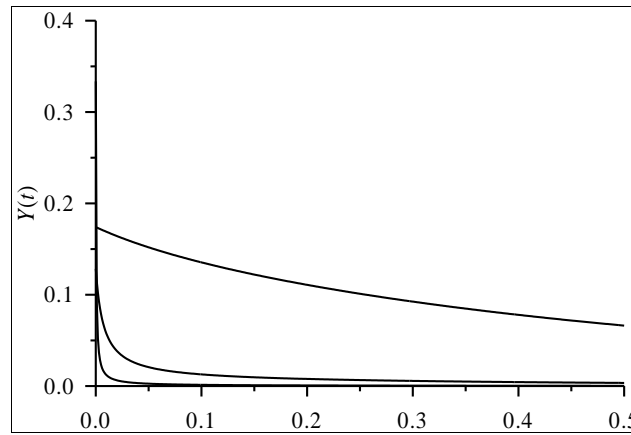
**Fig 1:** Dependences of the volume of products shipped from time for different values of parameters  $a_i$  and  $b_i$  with a constant increasing of volume of production  $d_0$  and delays  $\tau$



**Fig 2:** Dependences of the volume of the product being shipped on the value of the parameters  $a_i$  for various values of parameters  $b_i$ , at different moments of time  $t$  and at a constant increasing of volume of production  $d_0$  and delays  $\tau$



**Fig 3:** Dependences of the volume of the product being shipped on the value of the parameters  $b_i$  for various values of the parameters  $a_i$ , at different moments of time  $t$  and with a constant increasing of volume of production  $d_0$  and delays  $\tau$



**Fig 4:** Dependences of the volume of products shipped from the value of the delays  $\tau_i$  for different values of the parameters  $a_i$  and  $b_i$ , at different moments of time  $t$  and with a constant value of increasing of the volume of production  $d_0$

### Conclusion

In this paper we introduce a model for forecasting the activity of industrial enterprises based on solving a differential equation. The introduced model gives a possibility to make prognosis of industrial activity of the industrial enterprises taking into account change of volume of manufacturing of production, and also in view of change of its shipment to the domestic market and export deliveries. We also introduce an analytical approach to solve the differential equation.

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